Dimensional Crossover by a Local Inhomogeneity in Soliton-Pair Nucleation *

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Soliton-pair nucleation rates $\Gamma = A\exp(-B)$ are studied in highly-biased sine-Gordon systems with a local inhomogeneity for both a thermal activation regime and a quantum tunneling regime. It is found that the local inhomogeneity strongly affects the nucleation rates by modifying the bias-dependence of the exponent B. This change in the exponent B is explained as a dimensional crossover caused by the local inhomogeneity. It is also shown that there is another crossover, at which A becomes independent of the system size.

KEYWORDS: Sine-Gordon equation, soliton-pair nucleation, local inhomogeneity, long Josephson junctions

The dynamics of a one-dimensional sine-Gordon (SG) model has provided conventional understandings of various systems in physics, including dislocation in crystals, charge density waves (CDWs) in quasi-one-dimensional materials, and long Josephson junctions. One of typical processes in these systems is nucleation of soliton-antisoliton pairs driven by an external force. The nucleation rates of soliton-pairs have been calculated in both a thermal activation regime and a quantum tunneling regime for homogeneous systems.[1,2,3,4] Inhomogeneities, however, are unavoidable in actual experimental situations. They may change the nucleation process drastically, because the nucleation rate $\Gamma = Ae^{-B}$ is very sensitive to local modulation in the exponent B, which is proportional to a barrier height. Especially, inhomogeneities may change the bias-dependence of the exponent B, which can be detected experimentally. This possibility has been pointed out first in the context of CDW systems.[5]

In this Letter, effects of a local inhomogeneity on soliton-pair nucleation are studied at a high driving bias. The nucleation rates are evaluated by Langer's method.[7] The bias-dependence of B is typically expressed as $B \propto (f_{\rm c} - f)^\gamma$, where f and $f_{\rm c}$ are an external bias and a classical threshold bias, respectively. It is shown that the inhomogeneity changes not only the threshold current $f_{\rm c}$, but also the exponent γ. This change of γ affects the nucleation rates drastically, and is expected to be detected experimentally. It is claimed that the change of γ is essentially understood by a *dimensional crossover* caused by the local inhomogeneity. Another crossover about the prefactor A is also discussed. Although the following discussion is available for various physical systems described by the SG model, long Josephson junctions (LJJs) are considered as a comprehensive example, which allows well-controlled experiments.

The classical equation of motion of LJJs with a local impurity is given as[6]

$$\phi_{tt} - \phi_{xx} + \sin\phi - f - \varepsilon\delta(x)\sin\phi = 0, \tag{1}$$

where f is an external current density normalized by the critical current density, and $\varepsilon(>0)$ is a strength of an impurity potential made by modifying the thickness of insulator layers locally. Here, the spatial and temporal variables are normalized by the Josephson length and plasma frequency in LJJs. In this Letter, dissipation due to quasi-particle currents is assumed to be small, but not extremely weak, to guarantee the thermal equilibrium in a metastable state. The partition function of this system is described by the imaginary-time path integrals as $Z = \int \mathcal{D}\phi(x,\tau)\exp(-S_{\rm E}/g^2)$, where

$$S_{\rm E}[\phi(x,\tau)] = \int_{-L/2}^{L/2} {\rm d}x \int_0^{g^2/T} {\rm d}\tau \left[\frac{\phi_x^2}{2} + \frac{\phi_\tau^2}{2} - f\phi + (1-\varepsilon\delta(x))(1-\cos\phi) \right], \tag{2}$$

is the Euclidean action and g^2 is the normalized Planck constant. Here, L and T are the length of the junction and the temperature normalized by the Josephson energy per unit length, respectively. At high biases $f = 1-\eta$ ($\eta \ll 1$), the potential energy is allowed to be expanded to a quadratic-plus-cubic form. By changing the field variable as $\phi(x) = \pi/2 + \sqrt{2\eta}(\varphi(x)-1)$, and by rescaling the spatial (temporal) variables as $x = (2\eta)^{-1/4}x'$ ($\tau = (2\eta)^{-1/4}\tau'$), the Euclidean action is modified as

$$S_{\rm E}[\varphi(x',\tau')] = (2\eta)^1 \int {\rm d}x' \int {\rm d}\tau' \left[\frac{\varphi_{x'}^2}{2} + \frac{\varphi_{\tau'}^2}{2} + \frac{\varphi^2}{2} - \frac{\varphi^3}{6} - \tilde{\varepsilon}\delta(x')\varphi \right]. \tag{3}$$

Here, $\tilde{\varepsilon} = \varepsilon(2\eta)^{-3/4}$ is an effective impurity strength, which determines the magnitude of the inhomogeneity effects on nucleation. It should be noted that the impurity effects can be controlled by the external current

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$f = 1-\eta$; the effective impurity strength $\tilde{\varepsilon} = \varepsilon(2\eta)^{-3/4}$ is enhanced if the current approaches the classical threshold current as $\eta \to 0$.

The nucleation rates are evaluated by Langer's method in terms of the imaginary part of the free energy $F = -T\ln Z$.[7,8,9,10] The partition function Z is evaluated by integrating out the field $\varphi(x',\tau')$ up to the second order of fluctuations around stationary solutions determined by $\delta S_{\rm E}/\delta\varphi = 0$. In the present case, there are two stationary solutions: one is a stable solution $\varphi_0(x',\tau')$ and the other one is a bounce solution $\varphi_{\rm B}(x',\tau')$ with one unstable mode. The evaluated partition function Z includes an imaginary part produced by the integration around the bounce solution $\varphi_{\rm B}(x')$. The nucleation rate is then related to the free energy through $\Gamma = 2f(T){\rm Im}F$. Here, $f(T)$ is a temperature-dependent factor, and takes 1 for $T < T_0$, and T_0/T for $T > T_0$, where T_0 is the crossover temperature between the thermal activation regime and the quantum tunneling regime.[9,10,11,12] The nucleation rate $\Gamma = A\exp(-B)$ is then obtained as

$$A = \frac{f(T)T}{g^2}\prod_{i=1}^{\infty}\left(\frac{\lambda_i^{(0)}}{\lambda_i^{(\rm B)}}\right)^{1/2}, \tag{4}$$

$$B = (S[\varphi_{\rm B}(x',\tau')] - S[\varphi_0(x',\tau')])/g^2. \tag{5}$$

Here, $\lambda_i^{(0)}$s ($\lambda_i^{(\rm B)}$s) are the frequencies of eigenmodes around φ_0 ($\varphi_{\rm B}$) obtained by solving the 'Schrödinger' equation

$$\begin{aligned}&[-\partial_{x'x'} - \partial_{\tau'\tau'} + (1-\varphi_{0,\rm B}(x',\tau'))]\,\psi(x',\tau')\\ &\quad = \lambda_i^{(0,\rm B)}\psi(x',\tau').\end{aligned} \tag{6}$$

If there is a zero-frequency Goldstone mode around the bounce solution ($\lambda_i^{(\rm B)} = 0$), this mode must be replaced by the translational mode by using Fadeev-Popov technique.[8]

Thermal activation regime. At high temperatures $T > T_0 \sim g^2\eta^{1/4}$, the bounce solution $\varphi_{\rm B}(x',\tau')$ is independent of the imaginary time τ', and the ImF method reproduces the Kramers-type nucleation rate $\Gamma = A\exp(-\Delta U/T)$.[13,14] The energy barrier ΔU is calculated as

$$\Delta U = U[\varphi_{\rm B}(x')] - U[\varphi_0(x')], \tag{7}$$

$$\frac{U[\varphi(x')]}{(2\eta)^{5/4}} = \int_{-\infty}^{\infty}{\rm d}x'\left[\frac{\varphi_{x'}^2}{2} + \frac{\varphi^2}{2} - \frac{\varphi^3}{6} - \tilde{\varepsilon}\varphi\delta(x')\right]. \tag{8}$$

The stationary solutions are obtained as $\varphi_{\rm B}(x') = \varphi(x';a_1)$ and $\varphi_0(x') = \varphi(x';a_2)$ respectively, where

$$\varphi(x';a) = \frac{3}{\cosh^2((|x'|+a)/2)}. \tag{9}$$

The values of a_1, a_2 ($a_1 < a_2$) depend on the effective impurity strength $\tilde{\varepsilon}$ through $\tilde{\varepsilon} = 6\alpha(1-\alpha^2)$ with $\alpha = \tanh(a/2)$. At the current satisfying $\tilde{\varepsilon} = \varepsilon(2\eta)^{-3/4} = 4/\sqrt{3}$, the stationary solutions disappear, and the energy barrier ΔU becomes zero. Hence, the classical threshold current is modified by the impurity from the homogeneous case $\eta_{\rm c} = 0$ as $\eta_{\rm c} = (\sqrt{3}\varepsilon/4)^{4/3}/2$.

The bias-dependence of the energy barrier ΔU obtained numerically for $\varepsilon = 0.1$ is shown by solid lines in Fig. 1. At biases far from the threshold bias ($\eta \gg \eta_{\rm c}$), the impurity effect is weak, since the effective impurity strength $\tilde{\varepsilon}$ is small. In this case, the energy barrier ΔU is proportional to $\eta^{5/4}$ as in the homogeneous case, and the inhomogeneity gives only a small correction of order of $\tilde{\varepsilon}(\ll 1)$ as

$$\Delta U = (2\eta)^{5/4}\left(\frac{24}{5} - 3\tilde{\varepsilon} + \mathcal{O}(\tilde{\varepsilon}^2)\right). \tag{10}$$

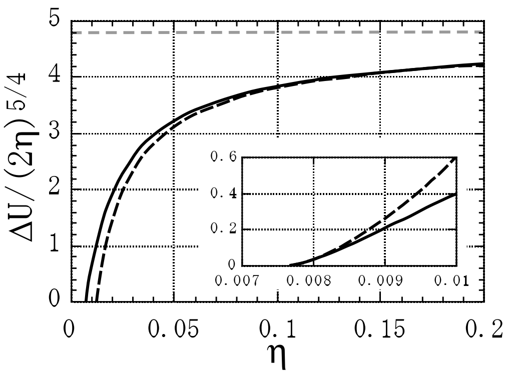

Fig. 1. The solid line denotes the energy barrier ΔU as a function of the bias current $\eta = 1-f$ obtained numerically for $\varepsilon = 0.1$. The dashed line denotes the result (10) for $\tilde{\varepsilon} = \varepsilon(2\eta)^{-3/4} \ll 1$, and the gray dashed line denotes that of the homogeneous case. The inset shows the behavior of ΔU near the threshold current $\eta = \eta_{\rm c}$ ($\eta_{\rm c} = 0.076$ for $\varepsilon = 0.1$), and the dashed line in the inset denotes the result (11) for $\tilde{\varepsilon} = 4/\sqrt{3} - \bar{\varepsilon}$ ($\bar{\varepsilon} \ll 1$).

This result is shown by a dashed line in Fig. 1. When η approaches the threshold bias as $\eta \to \eta_{\rm c}$, the effective impurity strength $\tilde{\varepsilon}$ approaches the threshold value $4/\sqrt{3}$, and the bounce solution is strongly modified. As a result, the bias-dependence of the energy barrier is modified. For the strong inhomogeneity $\tilde{\varepsilon} = 4/\sqrt{3} - \bar{\varepsilon}$ ($\bar{\varepsilon} \ll 1$), the energy barrier ΔU can be evaluated analytically as

$$\Delta U = \frac{8}{\sqrt{6}}(2\eta_{\rm c})^{5/4}\left(\frac{\eta-\eta_{\rm c}}{\eta_{\rm c}}\right)^{3/2} + \mathcal{O}\left(\frac{\eta-\eta_{\rm c}}{\eta_{\rm c}}\right)^{5/2}. \tag{11}$$

This result is shown by a dashed line in the inset of Fig. 1. Note that the exponent of the bias-dependence is changed from 5/4 (eq. (10)) to 3/2 (eq. (11)) by the strong inhomogeneity. This crossover about the exponent B at $\eta \sim 2\eta_{\rm c}$ is expected to be detected experimentally, and is explained by a dimensional crossover as discussed later.

The prefactor A is also affected by the local inhomogeneity through the change in the spectrum of $\lambda_i^{(0,\rm B)}$s. Generally, this change in the spectrum does not affect the prefactor so strongly as the change of the exponent. The change of the zero-frequency mode, however, may produce significant effects on A. In the homogeneous case, this zero-frequency mode around the bounce solution $\varphi_{\rm B}(x')$ produces the prefactor proportional to the system size L. In the presence of the inhomogeneity, this mode has a positive frequency, and modifies the L-dependence of A.

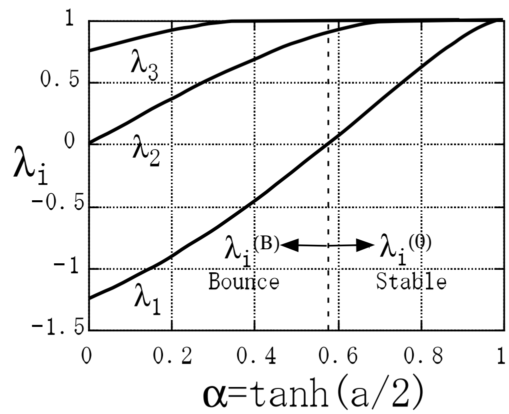

Fig. 2. The spectrum of eigenmodes around stationary solutions (9) is shown as a function of $\alpha = \tanh(a/2)$. For $\alpha < 1/\sqrt{3}$, it denotes the spectrum around the bounce solution $\varphi_{\rm B}(x')$, while for $\alpha > 1/\sqrt{3}$ around the stable solution $\varphi_0(x')$. In addition to these bound states, there exists continuum spectrum at $\lambda \geq 1$.

The spectrum of $\lambda_i^{(0,\rm B)}$s obtained numerically from the Scrödinger equation (6) is shown in Fig. 2. The region $\alpha > 1/\sqrt{3}$ corresponds to the stable solution, and $\alpha < 1/\sqrt{3}$ to the bounce solution. The lowest mode $\lambda_1^{(\rm B)}$ denotes the variable along the tunneling path. The second mode $\lambda_2^{(\rm B)}$ corresponds to the translational mode in the homogeneous case. As seen in Fig. 2, the frequency of the second mode remains small for the weak inhomogeneity $\tilde{\varepsilon} \ll 1$ ($\alpha \ll 1$). Hence, this mode must be treated carefully for $\tilde{\varepsilon} \ll 1$ by the Fadeev-Popov technique. Within the first-order perturbation for $\tilde{\varepsilon}$, the contribution of the second mode is treated as

$$\sqrt{\frac{1}{\lambda_2^{(B)}}} \rightarrow \sqrt{\frac{(2\eta)^{5/4}}{2\pi T}\frac{24}{5}} \int {\rm d}x_0 \exp\left[-\frac{\Delta U_{\rm imp}/T}{\cosh^2(Cx_0/2)}\right], \tag{12}$$

where $C = \sqrt{5/24}$. The energy modulation $\Delta U_{\rm imp} = 3\tilde{\varepsilon}(2\eta)^{5/4} = 3\varepsilon(2\eta)^{1/2}$ by the inhomogeneity determines the L-dependence. The prefactor A is proportional to L for $\Delta U_{\rm imp} \ll T$ ($\eta \ll \eta_{\rm cr}$) as in the homogeneous case. However, A becomes independent of L for $\Delta U_{\rm imp} \gg T$ ($\eta \gg \eta_{\rm cr}$). The crossover bias is estimated as $\eta_{\rm cr} \sim (T \ln L/\varepsilon)^2$. Note that this crossover at $\eta \sim \eta_{\rm cr}$ about A is independent of the crossover near $\eta \sim 2\eta_{\rm c}$ about the exponent B.

For the strong inhomogeneity $\tilde{\varepsilon} = 4/\sqrt{3} - \bar{\varepsilon}$ ($\bar{\varepsilon} \ll 1$), the frequency of the translational mode is so large that its contribution to the prefactor becomes independent of L. Only the frequencies of the lowest modes, $\lambda_1^{(\rm B)}$ and $\lambda_1^{(\rm B)}$, become small compared with the characteristic frequency of this system ($\lambda = 1$) and the other frequencies of the eigenmodes. Hence, only the lowest mode is relevant to the nucleation process. To clarify this situation, the field $\varphi(x')$ is truncated to a one-variable problem

$$\varphi(x') = \varphi(x'; a_{\rm c}) + C_1(\tau')\psi_1(x'; a_{\rm c}), \tag{13}$$

where $\varphi(x'; a)$ is given in (9), and $\psi_1(x'; a_{\rm c}) = C' \sinh((|x'| + a_{\rm c})/2)/\cosh^3((|x'| + a_{\rm c})/2)$ is the local deformation mode which becomes the zero-frequency mode at $a = a_{\rm c}$. The normalization constant C' is obtained from $\int {\rm d}x' |\psi_1|^2 = 1$ as $(135/(36 - 8\sqrt{3}))^{1/2}$, and the critical value $a_{\rm c}$ is determined by $\tanh(a_{\rm c}/2) = 1/\sqrt{3}$. The coefficient C_1 has dynamics in the τ-direction under the potential

$$U[\varphi(x')] = {\rm const.} + \frac{2\sqrt{3}}{9}\bar{\varepsilon}(C'C_1) - \frac{4}{243}(C'C_1)^3. \tag{14}$$

Thus, the nucleation is described by the one-variable potential produced by local deformation of the field. In this situation, the prefactor A cannot have the L-dependence, because the system size L is irrelevant to the nucleation process caused by the local deformation of the field $\varphi(x')$. Note that this potential form (14) reproduces the expression of the energy barrier (11) for the strong inhomogeneity.

The result is summarized in Fig. 3 for two possible cases. In the case of $\eta_{\rm cr} \gg \eta_{\rm c}$, the prefactor is proportional to L for the region $2\eta_{\rm c} < \eta < \eta_{\rm cr}$, and it is independent of L for the other current region. In the case of $\eta_{\rm cr} \ll \eta_{\rm c}$, the prefactor is independent of L for any η. The crossover at $\eta \sim \eta_{\rm cr}$ disappears because $\eta_{\rm cr}$ estimated for $\tilde{\varepsilon} \ll 1$ is not valid for the strong inhomogeneity $\eta < 2\eta_{\rm c}$ ($\tilde{\varepsilon} \sim 1$), where the prefactor A is independent of L for any η.

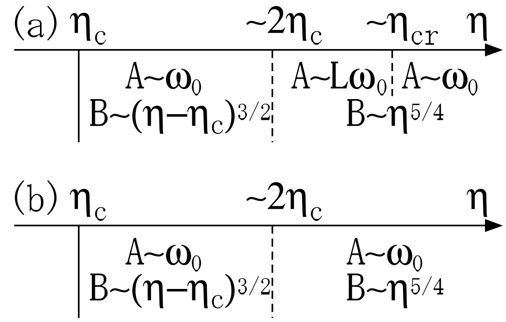

Fig. 3. The qualitative behaviors of the nucleation rate $\Gamma = Ae^{-B}$ (a) for $\eta_{\rm cr} \gg \eta_{\rm c}$ and (b) for $\eta_{\rm cr} \ll \eta_{\rm c}$. The characteristic attempt frequency is denoted with ω_0.

Quantum tunneling regime. At low temperatures $T < T_0 \sim g^2\eta^{1/4}$, nucleation due to quantum tunneling is dominant. In this regime, the stationary solutions of the action (3) must be calculated by solving the 1+1-dimensional classical field equation. However, as shown in the thermal activation regime, the features of the bias-dependence of the nucleation rate can be discussed by the perturbational treatment. Hence, in this Letter, only the limiting cases are discussed to clarify the bias-dependence of the nucleation rates.

For the weak inhomogeneity $\tilde{\varepsilon} \ll 1$, the exponent B is obtained within the first-order perturbation for $\tilde{\varepsilon}$ as

$$B = \frac{2\eta}{g^2}\left[s_0 + s_1\tilde{\varepsilon} + \mathcal{O}(\tilde{\varepsilon}^2)\right], \tag{15}$$

where $s_0 = 31.00$ and $s_1 = 16.43$. In this region, the exponent B is proportional to η^1 as in the homogeneous case, and the inhomogeneity effect only appears as a small correction. For the strong inhomogeneity $\tilde{\varepsilon} = 4/\sqrt{3} - \bar{\varepsilon}$ ($\bar{\varepsilon} \ll 1$), the system can be truncated

to the one-variable problem under the potential (14). In this region, the exponent B is obtained as

$$B \simeq \frac{15.8}{g^2}(\eta-\eta_c)^{5/4}. \tag{16}$$

Thus, the exponent of the bias-dependence is changed from 1 to 5/4 by the strong inhomogeneity at the crossover current $\eta \sim 2\eta_c$.

The system-size dependence of the prefactor A can be studied by a discussion parallel to that of the thermal activation regime. In the homogeneous case $\tilde{\varepsilon}=0$, there are two zero-frequency modes related to the spatial (temporal) translational symmetry of the bounce soliton $\varphi_B^{(0)}(x',\tau')$. In the weakly-inhomogeneous case $\tilde{\varepsilon} \ll 1$, the frequency of the temporal translational mode remains zero, while the frequency of the spatial translational mode is lifted. The Fadeev-Popov technique is applied to this spatial mode as

$$\sqrt{\frac{1}{\lambda_2^{(B)}}} \to \text{const.}\int \mathrm{d}x_0 \exp\left[-\frac{2\eta\tilde{\varepsilon}}{g^2}f(x_0)\right]. \tag{17}$$

Here, the function $f(x_0)=\int \mathrm{d}\tau' \varphi_B^{(0)}(x_0,\tau')$ behaves as $f(x_0)\to 0$ for $|x_0|\to\infty$, and has a maximum value 4.784 at $x_0=0$. It is found from (17) that the factor $\tilde{\varepsilon}\eta/g^2$ determines the system-size dependence of the prefactor A. For $\tilde{\varepsilon}\eta/g^2 \ll 1$ ($\eta \ll \eta_{cr}$), the prefactor is proportional to L, while for $\tilde{\varepsilon}\eta/g^2 \gg 1$ ($\eta \gg \eta_{cr}$), the prefactor becomes independent of L. The crossover bias is estimated as $\eta_{cr} \sim (g^2 \ln L/\varepsilon)^4$.

Discussion. The bias-dependence of the exponent B is summarized in Table I, where the results for small Josephson junctions (SJJs) are also shown for comparison. At high temperatures, the exponent B behaves for SJJs as $B \propto \eta^{3/2}$, and for homogeneous LJJs as $B \propto \eta^{5/4}$. This difference between SJJs and LJJs comes from the character of the bounce solution φ_B: the bounce solution has spatial dependence in the x'-direction for LJJs, while it is independent of x' for SJJs. In other words, there is a spatial dimensional crossover between SJJs and LJJs. Then, the bias-dependence of B is expressed as $B \propto \eta^{3/2-d/4}$ by a spatial dimension d which takes $d=1$ for LJJs and $d=0$ for SJJs. There also exists a 'temporal' dimensional crossover between the thermal activation regime and the quantum tunneling regime: the bounce solution φ_B has a temporal dependence in the τ'-direction at low temperatures, while it is independent of τ' at high temperatures. As a result, the bias-dependence of B at low temperatures is expressed as $B \propto \eta^{3/2-(d+1)/4}$, where $(d+1)$ denotes the total dimension of the system including the τ-direction.

The inhomogeneity effects on the bias-dependence of B can be understood in terms of a *dimensional crossover*. The inhomogeneity modifies both the bounce solution $\varphi_B(x',\tau')$ and the stable solution $\varphi_0(x',\tau')$. Although these two solutions have a spatial dependence, the difference between these solutions becomes small in the presence of the strong inhomogeneity. As a result, the nucleation process is effectively described by local deformation of the field, and the spatial dimension d is effectively reduced from $d=1$ to $d=0$ by the strong inhomogeneity.

	high-T	low-T
homogeneous LJJ	$B \propto \eta^{\frac{3}{2}-\frac{1}{4}}/\mathrm{T}$	$B \propto \eta^{\frac{3}{2}-\frac{1}{4}-\frac{1}{4}}/g^2$
SJJ	$B \propto \eta^{\frac{3}{2}}/T$	$B \propto \eta^{\frac{3}{2}-\frac{1}{4}}/g^2$
inhomogeneous LJJ	$B \propto (\eta-\eta_c)^{\frac{3}{2}}/T$	$B \propto (\eta-\eta_c)^{\frac{3}{2}-\frac{1}{4}}/g^2$

Table I. The bias-dependence of the exponent B in three systems: homogeneous (or weakly-inhomogeneous) long Josephson junction (LJJ), small Josephson junction (SJJ), and LJJ with the strong inhomogeneity $\tilde{\varepsilon} \sim 1$.

Therefore, in strongly-inhomogeneous LJJs, the exponent of η in B is the same as that of SJJs. (See Table I.)

In this Letter, soliton-pair nucleation rates $\Gamma = A\exp(-B)$ have been studied for highly-biased sine-Gordon (SG) systems with a local inhomogeneity. It is found that the effective inhomogeneity strength $\tilde{\varepsilon} \sim \varepsilon(1-f)^{-3/4}$ controls the bias-dependence of the exponent B which is typically written as $B \propto (f_c - f)^{3/2-d/4}$ in the thermal activation regime and as $B \propto (f_c-f)^{3/2-(d+1)/4}$ in the quantum tunneling regime, where f_c and f are the classical threshold bias and the external bias, respectively. The spatial dimension d takes $d=1$ for the homogeneous or weakly-inhomogeneous one-dimensional SG systems ($\tilde{\varepsilon} \ll 1$), while it is reduced to $d=0$ for strongly-inhomogeneous SG systems ($\tilde{\varepsilon} \sim 1$). This change in B is expected to be detected experimentally, *e.g.* in LJJs.[15, 16] This phenomena would be available to evaluate inhomogeneity strength in real experimental systems. It is also found that there exists a different crossover bias η_{cr} at which the prefactor becomes independent of the system size L. These results are universal for systems where the nucleation process is most dominant. Details of the relevance to actual experiments will be presented elsewhere.

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